

## COLLIN COLLEGE EXPANDED GENERIC COURSE SYLLABUS

### COURSE INFORMATION

**Course Number:** MATH 2318

**Course Title:** Linear Algebra

**Credit Hours:** 3

**Lecture Hours:** 3

**Lab Hours:** 0

#### Prerequisite

MATH 2414 with a C or better.

#### Course Description

Introduces and provides models for application of the concepts of vector algebra. Topics include finite dimensional vector spaces and their geometric significance; representing and solving systems of linear equations using multiple methods, including Gaussian elimination and matrix inversion; matrices; determinants; linear transformations; quadratic forms; eigenvalues and eigenvector; and applications in science and engineering.

#### Textbook/Supplies

Onsite Courses: *Elementary Linear Algebra*, 8<sup>th</sup> Edition by Ron Larson ©2017, Cengage Learning.

Supplies: Graphing calculator required.

### STUDENT LEARNING OUTCOMES (SLO)

Upon completion of this course the students should be able to do the following:

1. Be able to solve systems of linear equations using multiple methods, including Gaussian elimination and matrix inversion. (Empirical/Quantitative Skills)
2. Be able to carry out matrix operations, including inverses and determinants. (Empirical/Quantitative Skills)
3. Demonstrate understanding of the concepts of vector space and subspace. (Communication Skills)
4. Demonstrate understanding of linear independence, span, and basis. (Critical Thinking, Communication Skills)
5. Be able to determine eigenvalues and eigenvectors and solve problems involving eigenvalues. (Empirical/Quantitative Skills)
6. Apply principles of matrix algebra to linear transformations. (Critical Thinking Skills)
7. Demonstrate application of inner products and associated norms. (Communication Skills)

## REQUIRED CORE OBJECTIVES FOR MATHEMATICS

As per the Texas Higher Education Coordinating Board, mathematics students must develop and demonstrate the following three required core objectives:

- Critical Thinking Skills - creative thinking, innovation, inquiry, and analysis, evaluation and synthesis of information.
- Communication Skills - effective development, interpretation and expression of ideas through written, oral and visual communication.
- Empirical and Quantitative Skills - manipulation and analysis of numerical data or observable facts resulting in informed conclusions.

## METHOD OF EVALUATION

### Course requirements

Attending lectures, completing assignments and exams.

### Course format

Lecture and guided practice.

A minimum of four proctored exams and a proctored comprehensive final exam will be given. The final exam must count at least as much as any regular exam. Homework and/or quizzes may be used in place of one exam or in addition to exams. The weight of each of these components of evaluation will be specified in the individual instructor's addendum to this syllabus. All out-of-class course credit, including home assignments, service-learning, etc. may not exceed 25% of the total course grade; thus, at least 75% of a student's grade must consist of proctored exams, and no student may retake any of these exams.

## COURSE POLICIES

*College-wide policies are pre-loaded into the Concourse Syllabi and are not duplicated in the Expanded Generic Syllabi for each course.*

*Instructor specific policies should be added to the Concourse Syllabus.*

## COURSE CONTENT

Proofs and derivations will be assigned at the discretion of the instructor. The student will be responsible for knowing all definitions and statements of theorems for each section outlined in the following modules.

## Module 1

The student will be able to:

1. Recognize a linear equation in  $n$  variables. SLO 1
2. Identify a parametric representation of a solution set. SLO 1
3. Determine whether a system of linear equations is consistent or inconsistent. SLO 1
4. Use back-substitution and Gaussian elimination to solve a system of linear equations. SLO 1
5. Determine the size of a matrix and write an augmented or coefficient matrix from a system of linear equations. SLO 1
6. Use matrices and Gaussian elimination with back-substitution to solve a system of linear equations. SLO 1
7. Use matrices and Gauss-Jordan elimination to solve a system of linear equations. SLO 1 & 2
8. Solve a homogeneous system of linear equations. SLO 1
9. Set up and solve a system of equations to fit a polynomial function to a set of data points, as well as to represent a network. SLO 1
10. Add, subtract matrices, and multiply a matrix by a scalar. SLO 2
11. Multiply two matrices. SLO 2
12. Use matrices to solve a system of linear equations. SLO 2
13. Use properties of matrix operations to solve matrix equations. SLO 2
14. Identify the transpose of a matrix and compute the inverse of a matrix (if it exists). SLO 2
15. Use an inverse matrix to solve a system of linear equations. SLO 1 & 2
16. Factor a matrix into a product of elementary matrices. SLO 2
17. Determine and use an  $LU$ -factorization of a matrix to solve a system of linear equations. SLO 1 & 2
18. Use a stochastic matrix to measure consumer preference (optional).
19. Use matrix multiplication to encode and decode messages. SLO 2
20. Use matrix algebra to analyze Leontief input-output models (optional).
21. Use the method of least squares to determine the least squares regression line for a set of data (optional).
22. Compute the determinants of a  $2 \times 2$  matrix and a triangular matrix. SLO 2
23. Identify the minors and cofactors of a matrix and use expansion by cofactors to calculate the determinant of a matrix. SLO 2
24. Use elementary row and column operations to evaluate the determinant of a matrix. SLO 2
25. Recognize conditions that yield zero determinants. SLO 2
26. Calculate the determinant of a matrix product and a scalar multiple of a matrix. SLO 2
27. Calculate the determinant of an inverse matrix and recognize equivalent conditions for a nonsingular matrix. SLO 2
28. Calculate the determinant of the transpose of a matrix. SLO 2
29. Compute the adjoint of a matrix and use it to calculate its inverse. SLO 2
30. Use Cramer's Rule to solve a system of linear equations. SLO 2
31. Use determinants to determine the area, volume, and the equations of lines and planes. SLO 2

## Module 2

The student will be able to:

1. Represent a vector as a directed line segment. SLO 3
2. Perform basic vector operations in  $R^2$ . SLO 3
3. Perform basic vector operations in  $R^n$ . SLO 3
4. Write a vector as a linear combination of other vectors. SLO 3
5. Define a vector space and recognize some important vector spaces. SLO 3
6. Show that a given set is not a vector space. SLO 3
7. Determine whether a subset  $W$  of a vector space  $V$  is a subspace of  $V$ . SLO 3
8. Determine subspaces of  $R^n$ . SLO 3
9. Write a linear combination of a set of vectors in a vector space  $V$ . SLO 1 & 3
10. Determine whether a set  $S$  of vectors in a vector space  $V$  is a spanning set of  $V$ . SLO 2 & 4
11. Determine whether a set of vectors in a vector space  $V$  is linearly independent. SLO 1 & 4
12. Recognize bases in the vector spaces  $R^n$ ,  $M_{m,n}$ , and  $P_n$ . SLO 2 & 4
13. Identify the dimension of a vector space. SLO 1 & 3
14. Derive a basis for the row, a basis for the column space, and the rank of a matrix. SLO 2 & 4
15. Determine the nullspace of a matrix. SLO 2 & 3
16. Compute the solution of a consistent system  $A\mathbf{x} = \mathbf{b}$  in the form  $\mathbf{x}_p + \mathbf{x}_h$ . SLO 1 & 2
17. Determine a coordinate matrix relative to a basis in  $R^n$ . SLO 2 & 4
18. Calculate the transition matrix from the basis  $B$  to the basis  $B'$  in  $R^n$ . SLO 2 & 4
19. Represent coordinates in general  $n$ -dimensional spaces. SLO 2 & 3
20. Determine whether a function is a solution of a differential equation and calculate the general solution of a given differential equation (optional).
21. Use the Wronskian to test a set of solutions of a linear homogeneous differential equation for linear independence (optional).
22. Identify and sketch the graph of a conic or degenerate conic section and perform a rotation of axes (optional).
23. Compute the length of a vector and find a unit vector. SLO 7
24. Determine the distance between two vectors. SLO 7
25. Calculate a dot product and the angle between two vectors, determine orthogonality and verify the Cauchy-Schwarz Inequality, the triangle inequality, and the Pythagorean Theorem. SLO 7
26. Determine whether a function defines an inner product and calculate the inner product of two vectors in  $R^n$ ,  $M_{m,n}$ ,  $P_n$  and  $C[a, b]$ . SLO 7
27. Determine an orthogonal projection of a vector onto another vector in an inner product space. SLO 7
28. Show that a set of vectors is orthogonal and forms an orthonormal basis and represent a vector relative to an orthonormal basis. SLO 4 & 7
29. Apply the Gram-Schmidt orthonormalization process. SLO 4 & 7
30. Calculate the cross product of two vectors in  $R^3$  (optional).
31. Compute the linear or quadratic least squares approximation of a function (optional).
32. Determine the  $n$ th-order Fourier approximation of a function (optional).

## Module 3

The student will be able to:

1. Identify the image and preimage of a function. SLO 6
2. Show that a function is a linear transformation and find a linear transformation. SLO 6
3. Determine the kernel of a linear transformation. SLO 1 & 6
4. Derive a basis for the range, the rank, and the nullity of a linear transformation. SLO 4 & 6
5. Determine whether a linear transformation is one-to-one or onto. SLO 1 & 4 & 6
6. Determine whether two vector spaces are isomorphic. SLO 3 & 4 & 6
7. Identify the standard matrix for a linear transformation. SLO 2 & 6
8. Determine the standard matrix for the composition of a linear transformations and find the inverse of an invertible linear transformation. SLO 2 & 6
9. Compute the matrix for a linear transformation relative to a nonstandard basis. SLO 2 & 4 & 6
10. Determine and use a matrix for a linear transformation. SLO 2 & 6
11. Show that two matrices are similar and use the properties of similar matrices. SLO 2 & 6
12. Identify linear transformations defined by reflections, expansions, contractions, or shears in  $R^2$  (optional).
13. Use a linear transformation to rotate a figure in  $R^3$  (optional).
14. Verify eigenvalues and corresponding eigenvectors. SLO 2 & 5
15. Compute the eigenvalues and corresponding eigenspaces. SLO 1 & 2 & 5
16. Use the characteristic equation to determine eigenvalues and eigenvectors and calculate the eigenvalues and eigenvectors of triangular matrix. SLO 1 & 2 & 5
17. Identify the eigenvalues of similar matrices, determine whether a matrix is diagonalizable, and find a matrix  $P$  such that  $P^{-1}AP$  is diagonal. SLO 2 & 5
18. Derive, for a linear transformation  $T: V \rightarrow V$ , a basis  $B$  for  $V$  such that the matrix for  $T$  relative to  $B$  is diagonal. SLO 4 & 5 & 6
19. Recognize, and apply properties of symmetric and orthogonal matrices. SLO 2 & 5
20. Determine an orthogonal matrix  $P$  that orthogonally diagonalizes a symmetric matrix  $A$ . SLO 2 & 5 & 7
21. Use a matrix equation to solve a system of first-order linear differential equations (optional).
22. Calculate the matrix of quadratic form and use the Principal Axes Theorem to perform a rotation of axes for a conic and a quadratic surface (optional).
23. Solve a constrained optimization problem (optional).