DERIVATIVES - Basic Prosperities/Formulas/Rules for Calculus with Applications

If $f(x)$ and $g(x)$ are differentiable functions, c and n are any real numbers, then:

	Derivative	**Example**
Constant Rule	$(c)' = 0$, c is any constant	$(5)' = 0$
Power Rule	$(x^n)' = nx^{n-1}$, n is any number	$(x^5)' = 5x^4$ $(x)' = 1x^0 = 1$ $(\sqrt{x})' = \left(x^{\frac{1}{2}}\right)' = \frac{1}{2\sqrt{x}}$
Constant times a Function	$(cf)' = cf'$, c is any constant	$(5x^3)' = 5\cdot(x^3)' = 5\cdot 3x^2 = 15x^2$
Sum or Difference Rule	$(f \pm g)' = f' \pm g'$	$(x^3 - 6x^2)' = (x^3)' - (6x^2)' = 3x^2 - 6\cdot 2x = 3x^2 - 12x$
Product Rule	$(fg)' = f'g + fg'$	$y = (x^3+7)(4-x^2)$ $y' = (x^3+7)'(4-x^2) + (x^3+7)(4-x^2)'$ $y' = 3x^2(4-x^2) + (x^3+7)(-2x)$ $y' = -5x^4 + 12x^2 - 14x$
Quotient Rule	$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$	$y = \frac{3x+2}{5-2x}$ $y' = \frac{(3x+2)'(5-2x) - (3x+2)(5-2x)'}{(5-2x)^2}$ $y' = \frac{3(5-2x) - (3x+2)(-2)}{(5-2x)^2}$ $y' = \frac{19}{(5-2x)^2}$
Chain Rule	$[f(g(x))]' = f'(g(x))g'(x)$	$y = (x^2-7)^{10}$ $y' = 10(x^2-7)^9 \cdot (x^2-7)'$ $y' = 10(x^2-7)^9(2x)$ $y' = 20x(x^2-7)^9$

	Derivative	**Example**
Exponential Function	$\left(a^x\right)' = (\ln a)a^x$ $\left(e^x\right)' = (\ln e)e^x = 1\cdot e^x = e^x$	$\left(2^x\right)' = (\ln 2)2^x$ $\left(e^x\right)' = e^x$
Chain Rule Variants	$\left(a^{g(x)}\right)' = (\ln a)a^{g(x)}g'(x)$ $\left(e^{g(x)}\right)' = e^{g(x)}\cdot g'(x)$	$y = 5^{7x^2+1}$ $y' = (\ln 5)\left(5^{7x^2+1}\right)\left(7x^2+1\right)'$ $y' = (\ln 5)\left(5^{7x^2+1}\right)14x$ $y' = 14x(\ln 5)\left(5^{7x^2+1}\right)$ $\left(e^{5x}\right)' = e^{5x}\cdot(5x)' = 5e^{5x}$
Logarithmic Function	$\left(\log_a\lvert x\rvert\right)' = \dfrac{1}{(\ln a)x}$ $\left(\ln\lvert x\rvert\right)' = \dfrac{1}{x}$	$\left(\log_4\lvert x\rvert\right)' = \dfrac{1}{(\ln 4)x}$ $\left(\ln\lvert x\rvert\right)' = \dfrac{1}{(\ln e)x} = \dfrac{1}{x}$
Chain Rule Variants	$\dfrac{d}{dx}\left(\log_a g(x)\right) = \dfrac{g'(x)}{(\ln a)g(x)}$ $\dfrac{d}{dx}\left(\ln g(x)\right) = \dfrac{g'(x)}{g(x)}$	$\log_5(10x) = \dfrac{(10x)'}{(\ln 5)10x} = \dfrac{10}{(\ln 5)10x} = \dfrac{1}{(\ln 5)x}$, $x>0$ $\ln\left\lvert x^2+3\right\rvert = \dfrac{\left(x^2+3\right)'}{x^2+3} = \dfrac{2x}{x^2+3}$