Gauss-Jordan Elimination

To solve a matrix using Gauss-Jordan elimination, go column by column.	Getting 1's: • Row swap (*you can only do a row swap on your first step) [option 1 in row ops]				
First, get a 1 in the first row of the first column.					
1	 Multiplying row by reciprocal. [option 3 in row ops] 				
Then, get zeros as the remaining entries of that column. The order in which you get the remaining zeros does not matter.	 <u>Getting 0's:</u> Multiply by additive inverse and/or add rows. [option 2 or 4 in row ops] 				
	Three Possible Outcomes (Examples):				
	1) One Solution:				
0 Once this is done, your first column is complete. To begin your second column, get a 1 in the	1 0 0 5 0 1 0 2 0 0 1 4				
second row.	x = 5, y = 2, and z = 4. The solution is (5, 2, 4).				
	2) Infinite Solutions (dependent system):				
Then, get zeroes as the remaining entries of that column.	1 0 2 3 0 1 -3 4 0 0 0 0				
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	The equations are: x + 2z = 3 and $y - 3z = 4$				
For the last column, we begin by getting a 1 in the last row.	Solve for x and y: x = 3 - 2z and $y = 4 + 3z$				
	Thus, the solution is: (3 - 2z, 4 + 3z, z) with $z = any$ real number				
	3) No Solution (inconsistent system):				
We then proceed by getting zeroes as the remaining entries.	1 0 0 5 0 1 0 2 0 0 0 4				
	Here, this matrix tells us that 0 = 4 which we know is false. Thus, there is no solution.				

Example:

Solve the system represented by the matrix:	3	4	2	1
	1	2	3	4
	2	3	1	4

The first step is to get a 1 in the first row of the first column.

Since there is a 1 in the second row, we'll do a row swap. [Rowops: option 1]

 $R_1 \stackrel{SWA}{\longleftarrow}$

4

3

3 2 The second step is to get zeros in the remaining cells of the first column.

We will do this by using additive inverses. [Rowops: option 4]

[Rowops: option 4]

$\stackrel{AP}{\rightarrow} R$	2	-3R	$R_1 + I_2$	$R_2 - \frac{YIE}{2}$	LDS TH	HE NEW	$\stackrel{\prime}{\rightarrow} R_2$	-2 <i>I</i>	$R_1 + R_1$	$R_3 - \frac{YIE}{2}$	LDS TH	HE NEW	R ₃
3	4		1	2	3	4		ĺ	1	2	3	4 -11	
	1		0	-2	-7	-11							
1	4		2	3	1	4			0	-1	-5	-4	

Now that we are done with the first column, we move to the second column. The next step is to get a 1 in the second row of the second column. **[Rowops: option 3]**

 $-\frac{1}{2}R_2 \xleftarrow{\text{YIELDS THE NEW}} R_2$ $\begin{vmatrix} 1 & 2 & 3 & | \\ 0 & 1 & 7/2 & | \\ 0 & -1 & -5 & | \\ -4 \end{vmatrix}$

We now get zeros in the remaining entries of column 2.

[Rowops: option 2]

$$R_{2} + R_{3} \xrightarrow{YIELDS THE NEW} R_{3}$$

$$\begin{vmatrix} 1 & 2 & 3 & | & 4 \\ 0 & 1 & 7/2 & | & 11/2 \\ 0 & 0 & -3/2 & | & 3/2 \end{vmatrix}$$

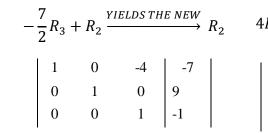
[Rowops: option 4]

Now that we are done with the second column, we move to the third column. We begin by getting a 1 in the third row. **[Rowops: option 3]**

$-\frac{2}{3}H$	$R_3 \xleftarrow{YIEL}$	DS THE N	$\xrightarrow{VEW} R_3$
1	0	-4	-7
0	1	7/2	11/2
0	0	1	-1
			•

Finally, we get zeros in the remaining entries of column 3.

[Rowops: option 4]



[Rowops: option 4]

4	$R_{3} +$	$R_1 \frac{YIE}{}$	LDS THE	$\xrightarrow{NEW} R_1$
	1	0	0	-11
	0	1	0	9
	0	0	1	-1