



Steps for Converting a Repeating Decimal Into a Rational Number

To determine if a decimal number is either rational or irrational, observe whether the decimal has a repeating pattern.

Example 1:

Determine if $0.\overline{123}$ is a rational number. If it is, determine its ratio.

$0.123123123123\dots$ is a rational number because "123" is a repeating pattern. Since the number is rational, let us convert this number into an actual ratio.

Observe the number of digits that are repeating. In this case, there are three digits repeating. Thus, we are going to use the number 1,000 since it has 3 zeroes. If two digits were repeating, such as with the number 0.12121212 , then we would use 100. If one digit was repeating, such as 0.11111 , we would use 10.

Now, let the rational number equal a variable. In this case, we will let $x = 0.123123123\dots$

Then, calculate the following

$$1000x - x = 123.123123\dots - 0.123123$$

$$999x = 123$$

$$x = \frac{123}{999}$$

Thus, this is the ratio that represents the number $0.\overline{123}$

NOTE:

Irrational numbers have no repeating pattern. Let's look at the first 10 digits of pi.

$$\pi \approx 3.141592654$$

Observe that there is no pattern. Thus, this number is irrational.