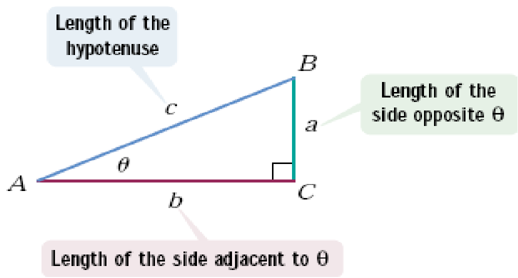
TRIGONOMETRY

Right Triangle Definition of Trig Functions



$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{a}{c} \qquad \csc\theta = \frac{hypotenuse}{opposite} = \frac{c}{a}$$

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{b}{c} \qquad \sec\theta = \frac{hypotenuse}{adjacent} = \frac{c}{b}$$

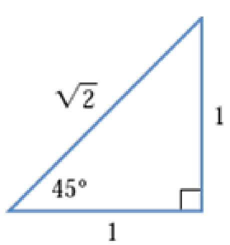
$$\tan\theta = \frac{opposite}{adjacent} = \frac{a}{b} \qquad \cot\theta = \frac{adjacent}{opposite} = \frac{b}{a}$$

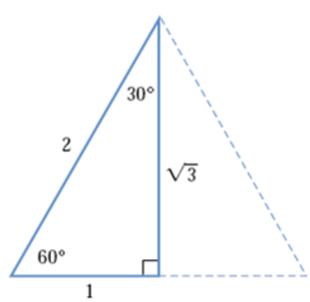
Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then:

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and } x = \frac{180t}{\pi}$$

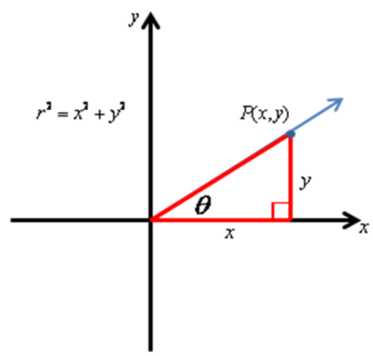
Sines, Cosines, and Tangents of Special Angles





θ	0°	$30^\circ = \frac{\pi}{6}$	$45^\circ = \frac{\pi}{4}$	$60^\circ = \frac{\pi}{3}$	$90^\circ = \frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	dne

Definitions of Trig Functions of Any Angle



Let θ be any angle in the standard position and let $P(x, y)$ be a point on the terminal side of θ. If r is the distance from $(0,0)$ to (x, y) and

$$r = \sqrt{x^2 + y^2} \neq 0, \quad x \neq 0, \quad y \neq 0$$

$$\sin\theta = \frac{y}{r}, \qquad \cos\theta = \frac{x}{r}, \qquad \tan\theta = \frac{y}{x}$$

$$\csc\theta = \frac{r}{y}, \qquad \sec\theta = \frac{r}{x}, \qquad \cot\theta = \frac{x}{y}$$

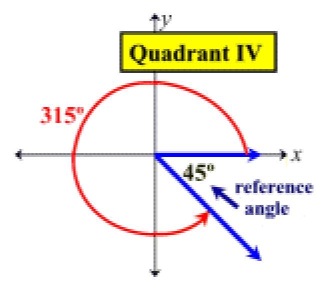
Reference Angles

Let θ be an angle in standard position. Its reference angle is the acute angle θ' formed by the terminal side of θ and the horizontal axis.

Example:

$\theta = 315^\circ$

$\theta' = 360^\circ - 315^\circ = 45^\circ$



The Signs of Trig Functions

	Q I	Q II	Q III	Q IV
$\sin\theta$	+	+	–	–
$\cos\theta$	+	–	–	+
$\tan\theta$	+	–	+	–

Formulas and Identities

Ratio: $\tan\theta = \dfrac{\sin\theta}{\cos\theta} \qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Even/Odd

$\cos(-\theta) = \cos\theta$ Even

$\left.\begin{array}{l}\sin(-\theta) = -\sin\theta \\ \tan(-\theta) = -\tan\theta\end{array}\right\}$ Odd

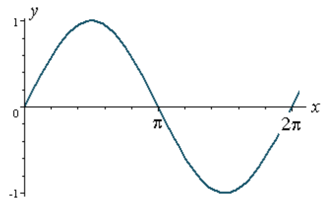
Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

TRIGONOMETRY

Graphs of Trig Functions



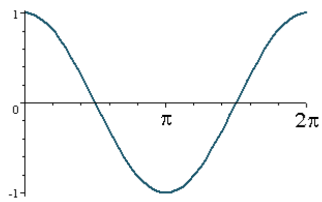
$y = \sin x$

Domain: $x \in (-\infty, \infty)$

Range: $y \in [-1, 1]$

Period: 2π

Amplitude: 1



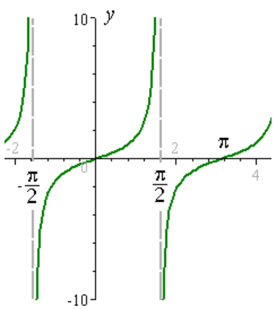
$y = \cos x$

Domain: $x \in (-\infty, \infty)$

Range: $y \in [-1, 1]$

Period: 2π

Amplitude: 1



$y = \tan x$

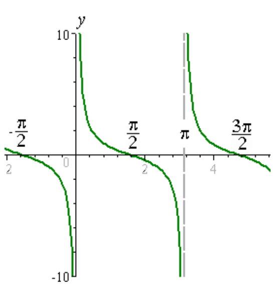
Domain:

$x \in \mathbb{R}$ and $x \neq \frac{\pi}{2} + n\pi$

Range: $y \in (-\infty, \infty)$

Period: π

Amplitude: None



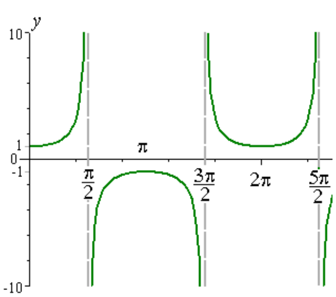
$y = \cot x$

Domain: $x \in \mathbb{R}$ and $x \neq n\pi$

Range: $y \in (-\infty, \infty)$

Period: π

Amplitude: None



$y = \sec x$

Domain:

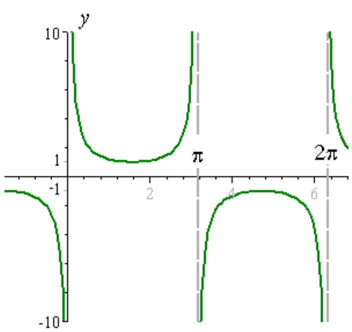
$x \in \mathbb{R}$ and $x \neq \frac{\pi}{2} + n\pi$

Range:

$y \in (-\infty, -1] \cup [1, \infty)$

Period: 2π

Amplitude: None



$y = \csc x$

Domain: $x \in \mathbb{R}$ and $x \neq n\pi$

Range:

$y \in (-\infty, -1] \cup [1, \infty)$

Period: 2π

Amplitude: None

Formulas and Identities

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half-Angle Formulas

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

Cofunction Formulas

$$\sin(90^\circ - \theta) = \cos\theta \qquad \cos(90^\circ - \theta) = \sin\theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

Inverse Trig Functions

$y = \sin^{-1} x$ is equivalent to $x = \sin y$, $\quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$y = \cos^{-1} x$ is equivalent to $x = \cos y$, $\quad 0 \le y \le \pi$

$y = \tan^{-1} x$ is equivalent to $x = \tan y$, $\quad -\frac{\pi}{2} < y < \frac{\pi}{2}$